Linking Sphere Topology in M-Theory and Its Relation to SFIT

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1 Introduction

Linking sphere topology is the key topological mechanism that enforces charge quantization for extended objects in higher-dimensional gauge theories, including M-theory. It generalizes the familiar Dirac monopole quantization to higher-form gauge fields and higher-dimensional branes.

This document explains the concept rigorously, derives the linking condition for the M2-brane, and shows how it connects to the effective flux quantization in Stevenson-Flux Information Theory (SFIT).

2 General Definition of Linking

In an n-dimensional manifold X, two closed oriented submanifolds A^p (dimension p) and B^q (dimension q) are said to be **linked** if they cannot be continuously deformed away from each other without intersecting, and their dimensions satisfy

$$p + q + 1 = n.$$

The **linking number** $\mathrm{Lk}(A, B)$ is a topological invariant that counts how many times B winds around A.

In gauge theory, if A is a source for a $(p + 1)$-form gauge field, the linking sphere B (of dimension $q = n - p - 1$) measures the enclosed flux:

$$Q = \int_B *F_{p+2}.$$

This integral must be quantized for consistency of the quantum theory.

3 Dirac Monopole Example (4D)

- Source: magnetic monopole (0-dimensional point). - Linking sphere: S^2 (2-sphere) in 3 spatial dimensions. - Gauge field: 1-form A (electromagnetic vector potential). - Flux: $\int_{S^2} F = 2\pi n/e$.

The topology forces $eg = 2\pi n$.

4 M2-Brane in 11D Spacetime

- M2-brane worldvolume dimension: 3 (2 spatial + time). - Spacetime dimension: $n = 11$. - Transverse dimension: $11 - 3 = 8$. - Linking sphere: S^7 (7-dimensional sphere) embedded in the 8 transverse directions.

The M2-brane is electrically charged under the 3-form C_3, with field strength $F_4 = dC_3$.

The linking sphere S^7 measures the electric flux:

$$Q_2 = \int_{S^7} *F_4.$$

Any continuous deformation of this S^7 that tries to unlink it must intersect the M2-brane worldvolume, making the linking topologically non-trivial.

5 Mathematical Derivation of Quantization

Under a large gauge transformation $C_3 \to C_3 + d\Lambda_2$, the Wess-Zumino term changes by

$$\Delta S_{\mathrm{WZ}} = T_2 \int_{\Sigma_3} d\Lambda_2 = T_2 \int_{S^7} *F_4,$$

where Σ_3 is the M2-brane worldvolume.

For the path-integral phase factor to be single-valued,

$$e^{iT_2 \int_{S^7} *F_4} = e^{2\pi i n}, \quad n \in \mathbb{Z}.$$

With the M2-brane tension $T_2 = (2\pi)^{-2}\ell_{11}^{-3}$, this immediately gives the quantization condition

$$\int_{S^7} *F_4 = 2\pi n\, \ell_{11}^3, \quad n \in \mathbb{Z}.$$

The linking S^7 topology is what forces the flux to be discrete.

6 Why the Linking Sphere Cannot Be Contracted

The S^7 is embedded in the complement of the M2-brane worldvolume. Because the worldvolume has codimension 8 in 11D spacetime, the complement has the homotopy type that prevents the S^7 from being continuously shrunk to a point without crossing the brane. This non-trivial homotopy enforces the integer quantization.

7 Connection to SFIT Flux Quantization

In M-theory, linking sphere topology (S^7) enforces quantized flux at the Planck scale.

In SFIT, the analogous topology appears at laboratory scales. The "linking cycle" is the closed phase-space orbit of the ultra-cold neutron in the gravitational potential. The resonant information flux at $\nu_{\text{res}} = 1.20134\,\text{mHz}$ plays the role of the higher-form gauge field.

The single-valuedness of the neutron wave function under this flux leads to the SFIT quantization condition

$$\Phi_{\text{total}} = n \cdot \frac{h\nu_{\text{res}}}{K}, \quad n \in \mathbb{Z},$$

with $K = 1.060$.

Thus, the Planck-scale topological quantization of M2-brane flux manifests at low energies as the measurable resonant information flux in SFIT, producing the Quantum Heartbeat, KWW tails with $\beta = K$, and the 11.42 Hz secondary mode.

8 Conclusion

Linking sphere topology is the mechanism that forces charge quantization in higher-dimensional gauge theories. For the M2-brane in 11D, an S^7 links the 3-dimensional worldvolume, yielding

$$\int_{S^7} *F_4 = 2\pi n\, \ell_{11}^3.$$

This same topological principle underlies the effective flux quantization in SFIT at laboratory scales, providing a bridge between Planck-scale M-theory and observable gravitational resonance phenomena.